Non-Archimedean Quantum Mechanics

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論 文 内 容 要 旨

Physical motivation. It is well known that, traditionally, real numbers are used in theoretical and mathematical physics because length of segments, size of angles and etc. should be measured precisely from the Archimedean axioms. However, for instance, the real space-time $\mathbf{R}^4$ seems still to be not adequate to deal with certain microscopical and cosmological phenomena, and in quantum gravity and in string theory it was proved that a measurement of distance smaller than the Planck length (approximately$10^{-33}cm$) is impossible. Thus the non-Archimedean (n.a.) structure of space-time in quantum physics was considered by Blij and Monna [BM 68]. This paper did not find any response in physics and has been forgotten. Vladimirov and Volovich [VV 84] considered supersymmetric models on superspaces over the n.a. locally compact fields. The n.a. physical models provoked great interest in connection with string theory (see, for instance, [Vol 87a, 87b]). However, it proved to be very difficult to interpret physical models of a level as high as a p-adic string. Therefore, simpler n.a. models such as quantum mechanics and field theory were investigated : In [Vla 88] the theory of generalized functions, the p-adic Gaussian integrals based on the real valued Haar measure, and the theory of Fourier transformations were studied over the field $\mathbf{Q}_p$ of p-adic numbers. Since these theories, two formalism of quantization over the space $\mathbf{Q}_p^n$ of were proposed in [VV 89]. The first approach considers wave function $f : \mathbf{Q}_p^n \to \mathbf{C}$, and the second approach considers function $f : \mathbf{Q}_p^n \to \mathbf{Q}_p$.

Basic of the theory of non-Archimedean quantum mechanics with C-valued functions. The formalism of the p-adic quantum mechanics with $\mathbf{C}$-valued functions is based on a triple

$$\{L^2(\mathbf{Q}_p^n), W(z), U(t)\},$$

where $\{L^2(\mathbf{Q}_p^n)$ is the Hilbert space of $\mathbf{C}$-valued square integrable functions on $\mathbf{Q}_p^n$ with respect to the real valued Haar measure, the family of unitary operators $W(z)$ determines a representation of the Heisenberg-Weyl group in $\{L^2(\mathbf{Q}_p^n)$, z is a point in the classical phase space $\mathbf{Q}_p^n \times \mathbf{Q}_p^n$, and the evolution operator $U(t)$, $t \in \mathbf{Q}_p$ gives a unitary representation of an additive subgroup of $\mathbf{Q}_p^n$ which defines dynamics. In particular, the quantum dynamics of the free particle and the harmonic oscillator were constructed. The above formalism was extended, by Zelenov [Zel 91, 92, 93, 94a, 94b], to the case of infinite-dimensional space, and the theory of representation of the p-adic Heisenberg group was studied. Zelenov's theory is based on a Weyl system (H, W) on the arbitrary dimensional p-adic sympletic space (V, B), where H is a complex Hilbert space and W is a continuous mapping from V to the family of unitary operators on H satisfying the condition

$$W(x)W(y) = \chi_p(B(x, y)/2)W(x+y).$$

Result 1. The p-adic Green function $G(x)$ was proposed in [VV 89] as follows :

$$(1.11) \qquad G(x) = \int_{\mathbf{Q}_p^n} \frac{\chi_p((k, x))}{|(k, k)|_p + m^2} dk, \quad m \in R_{>0},$$

where χ_p is an additive character of $\mathbf{Q}_p$ defined by $\chi_p(x) = \exp(2\pi\sqrt{-1}\{x\}_p)$, $\{x\}_p$ is the fractional part x, and $(k, x) = \Sigma_{j=1}^{n} k_j x_j$. Using Lebesgue's theorem, Fubini's theorem and Weierstrass' criterion, $G(x)$ is expressed as

$$(1.12) \qquad G(x) = \lim_{N,\, \varepsilon \to \infty} \int_0^{\varepsilon} \exp(-m^2\theta) \sum_{\alpha=0}^{\infty} \frac{(-\theta)^\alpha}{\alpha\,!} J(\alpha,\ n)\, d\theta,$$

where

$$(1.13) \qquad J = J(\alpha,\ n) = \int_{(p^{-N}\mathbf{Z}_p)^n} |(k,\ k)|_p^{\alpha}\ \chi_p((k,\ x))\, dk.$$

The properties of $G(x)$ for $n = 1$ were studied in [Vla 88]. For any odd prime p, the asymptotic expansions of $G(x)$ for $n = 2$ and for $n = 4$ were, respectively, given by Bikulov [Bik 91] and Kochubei [Koc 93].

In § 2, more generally, we obtain an asymptotic expansion of the p-adic Green function $G(x)$ for any even dimension n and any odd prime p (resp. for any even dimension n and $p = 2$) by calculating (1.13) in the functional equation of the local zeta function due to Rallis and Schiffmann [RS 73] (resp. in the t-representation due to Bikulov [Bik 91] and formulas of the p-adic Gaussian integral).

Result 2. First of all, we recall that the definition of *l-sheaf* on the *l-space* by Bernshtein and Zelevinskii [BZ 76, pp. 6 − 9] : A topological space X is said to be an *l-space* if it is Hausdorff, locally compact, and zero-dimensional. Denote by $C^\infty(X)$ and $S(X)$ the space of all locally constant $\mathbf{C}$-valued functions on X and the space of Schwartz-Bruhat functions on X, respectively. We say that an *l-sheaf is defined on* X if with each $x \in X$ there is associated a $\mathbf{C}$-vector space $\mathcal{F}_x$ and there is defined a family $\mathcal{F}$ of cross-sections (that is, mapping φ defined on X such that $\varphi(x) \in \mathcal{F}_x$ for each $x \in X$) such that the following conditions hold :

(i) $\mathcal{F}$ is invariant under addition and multiplication by functions in $C^\infty(X)$.

(ii) If φ is a cross-section that coincides with some cross-section in $\mathcal{F}$ in a neigh-bourhood of each point, then $\varphi \in \mathcal{F}$.

(iii) If $\varphi \in \mathcal{F}$, $x \in X$, and $\varphi(x) = 0$, then $\varphi = 0$ in some neighbourhood of x.

(iv) For any $x \in X$ and $\xi \in \mathcal{F}_x$ there exists a $\varphi \in \mathcal{F}$ such that $\varphi(x) = \xi$.

The l-sheaf itself is denoted by $(X,\ \mathcal{F})$. The spaces $\mathcal{F}_x$ are called *stalks.* and the elements of $\mathcal{F}$ *cross-sections of the sheaf.* We call the set supp $\varphi = \{x \in X : \varphi(x) \neq 0\}$ the support of the cross-section $\varphi \in \mathcal{F}$. Condition (iii) guarantees that supp φ is closed.

A cross-section $\varphi \in \mathcal{F}$ is called *finite* if supp φ is compact. We denote the space of finite cross-sections of $(X,\ \mathcal{F})$ by $\mathcal{F}_c$. It is clear that $\mathcal{F}_c$ is an $S(X)$-module, and that $S(X)\,\mathcal{F}_c = \mathcal{F}_c$. It turns out that this property can be taken as the basis for the definition of an l-sheaf.

Proposition 3.2.1 (cf.[BZ 76, Proposition 1.14]). *Let M be an $S(X)$-module such that $S(X)M = M$. Then there exists one and, up to isomorphism, only one l-sheaf* $(X,\ \mathcal{F})$ such that M is isomorphic as an S(X)-module to the space of finite cross-sections $\mathcal{F}_c$.

This means that defining an l-sheaf on X is equivalent to defining an $S(X)$-module M such that $S(X)M = M$.

In § 3, in order to define an l-sheaf over the finite-dimensional p-adic sympletic space

(V, B), we introduce the concept of an algebraic Weyl system (H, W) on (V, B), and some necessary and sufficient conditions for a Weyl system (H, W) to be irreducible are investigated. As application, we give another proof of the Ston-Von Neumann Theorem of the p-adic Heisenberg group. From the Schrödinger representation associated to a selfdual $\mathbf{Z}_p$-lattice $\mathcal{L}$ in (V, B), we construct a Weyl system $(H(\mathcal{L}, \sigma), W_{\mathcal{L}}, \sigma)$ depending a selfdual $\mathbf{Z}_p$-lattice $\mathcal{L}$ and a $\mathbf{Q}_p$-valued function σ.

論文審査の結果の要旨

数理物理学の１分野としてV.S. VladimirovやI.V. Volovichなどによるp−進体上での物理学の研究が有る。これらの研究は，物理学よりの要請により行われているが，数学的には量子力学などに出てくる特殊関数やポテンシャルを，p−進体の上の特殊関数やポテンシャルに置き換え，その数学的性質を研究することになる。張永浩（Jang Youngho）は東北大学の博士課程後期に編入後，一貫してこの分野の研究を行ってきた。

張は東北大学に入学前に位相幾何学やJordan代数などについて韓国内で３編の論文を発表しているが，東北大学理学研究科博士課程後期に編入後，さらに以下の２つの研究を行い，本論文として纏めた。

まず第1の研究では，数理物理からの要請の有ったp−進体$\mathbb{Q}_p$上の複素数体$\mathbb{C}$に値を取る積分（Green関数）を具体的に計算した。張は，直接計算によりこの積分の値を求めその漸近展開を求めたが，後に，この積分を直交群が作用する概均質ベクトル空間の局所ゼータ関数と見做し，その関数等式を使って証明をさらに簡易化した。

この結果は，Jang Youngho, An asymptotic expansion of the p-adic Green function, to appear in Tohoku Mathematical Journal.
として掲載されることが決まっている。

第２の研究においては，数理物理に出てくるp−進体$\mathbb{Q}_p$上の$\mathbb{C}$に値を取るp-adic Weyl system（E.I. Zelenov 他が定義）の性質を研究した。とくにp-adic Weyl systemの代数性の定義を与え，また代数的になるための判定条件を求めた。さらにp-adic Weyl systemが既約となるための必要十分条件を求め，それを使ってp-adic Heisenberg群についてのStoneとVon Neumannの定理の別証を与えた。

この結果は，既にJang-Youngho, Algebraic Weyl system and application, Ann. Math. Blaise Pascal, 4（1997）, 27-40.
として発表されている。

以上の張の研究は，p−進体の上での数理物理学の研究にとって非常に重要な研究であり，張が自立して研究を行うために必要な，高度な研究能力と学識を有することを示している。よって，張永浩提出の本論文は，博士（理学）の論文として合格と認める。